CALCULATION OF PARAMETERS OF A FLAT TURBULENT JET IN A DEFLECTING STREAM

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On the basis of the solution of the problem of a flat turbulent jet in a deflecting stream obtained earlier by the author a method is suggested for calculating the parameters of the main section of such a jet with allowance for the rarefaction occurring behind it. The amount of rarefaction was determined from an experiment and is accounted for by a second empirical constant. The results of a calculation by the proposed method are in satisfactory agreement with the experimental data.

The solution of the problem of a flat turbulent jet in a deflecting stream is presented in [1, 2]. The results obtained in these works show that when there is satisfactory agreement for the velocity u_m at the axis of the jet and for the ordinates δ_1 and δ_2 of its boundaries the axis of the jet is bent less than indicated by the experimental results. The suggestion was made in [2] that this difference between experiment and calculation is explained by the fact that rarefaction behind the jet was not taken into account. In the present report an attempt is made to allow for this rarefaction.

To a certain extent the flow in the zone behind the jet is analogous to the flow in the "dead" zone behind a plate mounted at right angles to the impinging stream. From physical considerations one should expect that the rarefaction behind a jet must be somewhat less than behind a plate. In fact, when a deflecting stream flows around a jet the return currents behind it are stimulated with less velocity than behind a plate. This occurs because the velocity in the jet which separates the "dead" zone from the rest of the flow decreases steadily, and if the return currents are intense then the rarefaction behind the jet must be somewhat less than behind a plate.

From the results of the experiment described in [2] the maximum relative rarefaction behind the jet was constructed as a function of the ratio of the initial velocity u_0 of the jet to the velocity v_∞ of the deflecting stream (Fig. 1). It turned out that this rarefaction is approximately constant and equal to $2\Delta p/\rho V_\infty^2 = 0.7$. The value of the rarefaction obtained in this way can be considered as a second empirical constant. The rarefaction found from the experiment was used in the calculation whose results are presented in Figs. 2 and 3. It is seen from a comparison with the experiment that with the introduction of a correction for the rarefaction the results of the calculation agree satisfactorily with the experimental data for all the parameters examined.

Values calculated on an electronic computer for the ordinate of the exterior (facing toward the stream) boundary ξ_e of the jet, the distance x along the axis of the jet, the radius of curvature R of the axis, and the relative velocity u_δ/V_∞ at the exterior boundary of the jet as functions of the coordinate ξ and the value

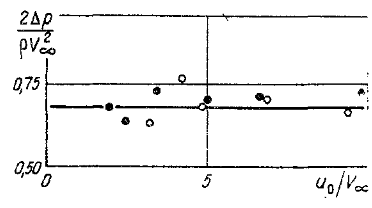

Fig. 1. Dependence of relative rarefaction on ratio of initial velocity of jet to velocity of deflecting stream (black circles were obtained in a study of the main section of the jet, open circles in a study of the initial section).

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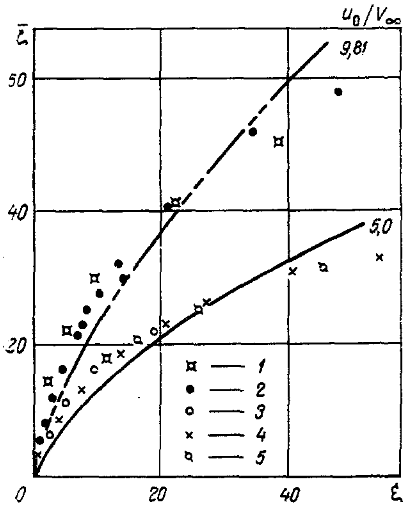

Fig. 2

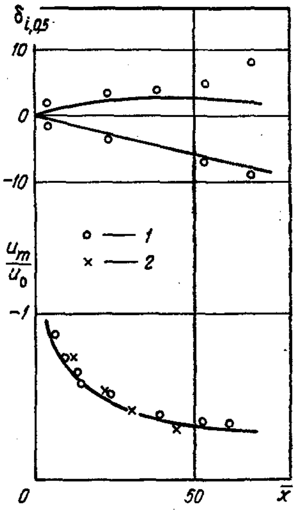

Fig. 3

Fig. 2. Comparison of calculated axis of jet with experimental data: solid line, calculation for $u_0/V_\infty = 5$; broken line, calculation for $u_0/V_\infty = 9.81$; 1) experiment [4] at $u_0/V_\infty = 10$); 2) [2] at $u_0/V_\infty = 9.81$; 3) [2] at $u_0/V_\infty = 5$; 4) [5] at $u_0/V_\infty = 5$; 5) [4] at $u_0/V_\infty = 5$.

Fig. 3. Comparison of calculated boundaries of jet and velocity at axis of jet obtained on electronic computer with experimental data at $u_0/V_\infty = 5$: solid lines; calculations: 1) experiment [2]; 2) [3].

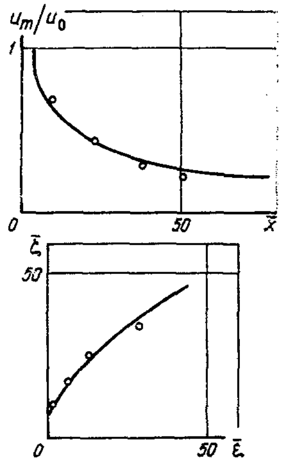

Fig. 4. Comparison of ordinates of jet and velocity at axis of jet calculated by proposed method with experimental data [3] ($u_0/V_\infty = 7.06$).

$K_2 = (u_0/V_\infty)^2 b_{02}$ are presented in Table 1. (Here (ξ, ζ) is an orthogonal coordinate system whose axes are directed along the direction of the deflecting stream (ξ) and along the direction of discharge of the jet (ζ), while the origin of the coordinates is located at the fictive source; b_{02} is the width of the interior downstream part of the nozzle cut relative to the width of the nozzle b_0. This quantity is determined from the solution for the initial section [2]. To a first approximation it can be set at 0.5.)

The calculations showed that the dependences presented in Table 1 are practically unchanged during variation of the empirical constant β in the range 0.08–0.1 usually encountered. The data presented in the table can be used for a simplified calculation of the parameters of a jet in a deflecting stream in the range of variation in u_0/V_∞ from 3.5 to 10. Such a simplified calculation can be organized in the following way. Through interpolation from Table 1, ζ_e, x, R, and u_δ/V_∞ as functions of ξ are determined for a given K_2. These are the auxiliary values for calculation by the equations presented in [1] and [2]. Since the dependence of δ_2 on x is close to linear, as seen from Fig. 3, one can assume for simplicity that this dependence is the same as that in a submerged jet, i.e.,

$$\delta_2 = -\frac{225}{8}\beta^2 x. \tag{1}$$

Thus, with the help of Eq. (1) and the dependence $x(\xi)$ found from Table 1 one can find the ordinate δ_2 of the interior boundary of the jet as a function of ξ. Then δ_1 and u_m/V_∞ are determined from the equations obtained in [1] for the values of δ_2 found using Eq. (1):

$$\left(\frac{u_m}{V_\infty}\right)^2 = \frac{K_2}{\delta_2\left(\frac{33}{112} - \frac{5}{84}\frac{\delta_2}{R}\right)}, \tag{2}$$

$$\delta_1 = -\delta_2\left(1 - \frac{u_\delta}{u_m}\right)^{2/3} \tag{3}$$

TABLE 1. Dependences of R, x, u_δ/V_∞, and ζ_e on ξ and K_2

ξ	R				x				u_δ/V_∞				ζ_e			
	K_2				K_2				K_2				K_2			
	4,13	11,35	20,9	48,1	4,13	11,35	20,9	48,1	4,13	11,35	20,9	48,1	4,13	11,35	20,9	48,1
0	13,5	28	50	100	0	0	0	0	0	0	0	0	0	0	0	0
5	21	37	57	122	11,2	18	21,5	32,5	0,945	0,640	0,470	0,352	10,4	16,4	22,5	32,5
10	39	50	67	142	17,5	25	32,6	48,2	1,105	0,835	0,640	0,500	14,1	23,2	30,7	45,3
15	69	59	77	160	23	32,2	41	58,8	1,150	0,959	0,765	0,600	16,6	28,0	37,3	56,5
20	91	72	87	173	28,2	38,8	47,2	68,5	1,175	1,030	0,855	0,683	18,5	31,5	42,8	65,2
25		85,5	98,5	187		44,5	54,3	77,6		1,068	0,907	0,748		34,9	47,1	73
30		97	112	198		50	61	86,1		1,100	0,940	0,798		37,5	51,2	80
35		111,5	125	208		55,7	67	95		1,115	0,965	0,850		40,0	54,7	86,4
40		128	140	216		61,2	73,5	102,8		1,125	0,982	0,900		42,0	57,8	92,0
45		146	158	223		66,9	79,1	110,2		1,132	0,995	0,932		44,0	60,6	96,1
50		164	179	232		71,8	83,5	117,0		1,135	1,000	0,965		45,7	62,7	98,6

To determine the axis of the jet one must plot the value of δ_1 from the external boundary along the normal to it. Here it is assumed that the normal to the external boundary of the jet differs little from the normal to the axis. As a result, all the parameters of the jet are obtained as functions of the coordinate ξ which is reckoned from the fictive source. In order to determine the connection between the parameters found and the nozzle cut one must find from the solution for the main section of the jet the coordinates of the cross section in which the velocity u_m at the axis is equal to the velocity u_0 at the nozzle cut, i.e., find the coordinates of the transition cross section. From Eq. (2), neglecting the small contribution from the term δ_2/R and using the condition $u_m = u_0$, we obtain an equation for the ordinate of the interior boundary of the jet at the transition cross section

$$\delta_{2t} = -\frac{K_2}{\frac{33}{112}(u_0/V_\infty)^2}. \tag{4}$$

From (1) we have

$$x_t = -\frac{8}{225}\delta_{2t}\beta^{-2}. \tag{5}$$

Knowing x_t one can find the corresponding values of ξ_t and ζ_t. If one assumes that the transition cross section coincides with the end of the initial section then one must decrease the values ζ_e, x, and ξ found by $(\zeta_t - \zeta_i)$, $(x_t - x_i)$, and $\xi_t - (\xi_i - \delta_{1t})$, respectively in order to find the corresponding coordinates reckoned from the nozzle cut. Here ξ_i and ζ_i are the coordinates of the end of the initial section and x_i is the distance along the axis of the jet from the nozzle cut to the last cross section of the initial section, which to a first approximation can be considered as the same as for a submerged jet and equal to (see [2])

$$x_i = \frac{35}{1248}\beta^{-2}.$$

To determine ξ_i and ζ_i the radius of curvature of the axis of the jet in the initial section is first calculated from the equation

$$R = 2\left[\ln\frac{(u_0/V_\infty)^2}{(u_0/V_\infty)^2 - 1}\right]^{-1}. \tag{6}$$

Following [2], one can assume that the radius of curvature of the axis of the jet is constant in the initial section. Then ξ_i and ζ_i represent the coordinates of a point on a circle having a radius R and a center which does not coincide with the origin of the coordinates, and they are determined from the equations

$$\xi_i = R\left(1 - \cos\frac{x_i}{R}\right),$$

$$\zeta_i = \sqrt{2R\xi_i - \xi_i^2}.$$

A calculation was made by the method described for $u_0/V_\infty = 7.06$ ($K_2 = 25$). Its results are presented in Fig. 4. It is seen from a comparison of the calculation with the experimental data of [3] that the calculation is in satisfactory agreement with experiment.

NOTATION

x, y	are curvilinear coordinates: x coincides with axis of jet and is reckoned from flat source, y is orthogonal to x and directed from axis of jet toward its external boundary;
ξ, ζ	are the longitudinal and transverse coordinates: ξ coincides with direction of deflecting of stream and is reckoned from flat source, ζ coincides with direction of emergence of jet;
δ_1, δ_2	are the ordinates of y at external and internal boundaries of jet;
b_{02}	is the ratio of width of internal (downstream) part of nozzle cut to width of nozzle;
$K_2 = (u_0/V_\infty)^2 b_{02}$	is the relative kinematic impulse of internal part of jet;
Δp	is the rarefaction behind jet;
ρ, u_0, V_∞	are the density, velocity of emergence of jet, and velocity of deflecting stream, respectively;
u_m, u_δ	are the velocity at axis of jet and at its external boundary;
R	is the radius of curvature of axis of jet;
β	is the empirical constant;
i	is the end of initial section;
t	is the transition cross section.

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